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CP PUBLICATION

SHORT TRICKS

Mathematics

for JEE (Main & Adv)

Sunil Gupta
Career Point Editorial

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SHORT TRICKS
Mathematics
for JEE (Main & Advanced)

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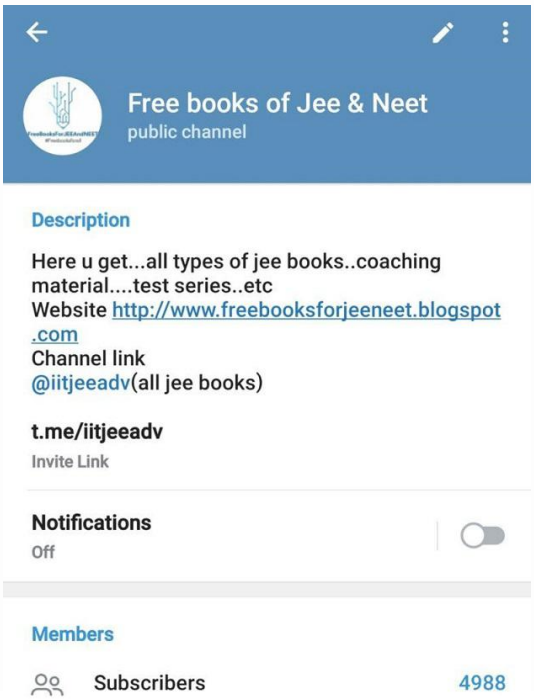

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14

Function

KEY CONCEPTS

1. Numbers and their Sets

(i) Natural Numbers : $N = \{1, 2, 3, 4,...\}$

(ii) Whole Numbers : $W = \{0, 1, 2, 3, 4,....\}$

(iii) Integer Numbers : I or $Z = \{...-3, -2, -1, 0, 1, 2, 3,...\}$

$Z^+ = \{1, 2, 3,....\}$, $Z^- = \{-1, -2, -3,....\}$

$Z_0 = \{\pm 1, \pm 2, \pm 3,....\}$

(iv) Rational Numbers : $Q = \{p/q ; p, q \in Z, q \neq 0\}$

(v) Irrational numbers : The numbers which are not rational or which can not be written in the form of p/q, called irrational numbers

eg. $\{\sqrt{2}, \sqrt{3}, 2^{1/3}, 5^{1/4}, \pi, e,....\}$

2. Interval

(i) Close interval $[a, b] = \{x, a \leq x \leq b\}$

(ii) Open interval (a, b) or $]a, b[= \{x, a < x < b\}$

(iii) Semi open or semi close interval

$[a, b[$ or $[a, b) = \{x; a \leq x < b\}$

$]a, b]$ or $(a, b] = \{x; a < x \leq b\}$

3. Function

Let A and B be two given sets and if each element $a \in A$ is associated with a unique element $b \in B$ under a rule f, then this relation is called Function.

Here, b is called the image of a and a is called the pre-image of b under f.

Note :

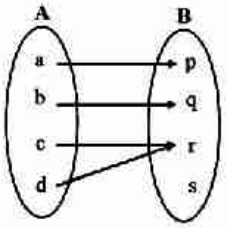
- Every element of A should be associated with B but vice-versa is not essential.
- Every element of A should be associated with a unique (one and only one) element of but any element of B can have two or more relations in A.

4. Domain, Codomain and Range of Function

If a function f is defined from a set of A to set B then for f: A→ B set A is called the **domain** of function f and set B is called the **co-domain** of function f. The set of the f- images of the elements of A is called the **range** of function f.

Domain = All possible values of x for which f(x) exists.

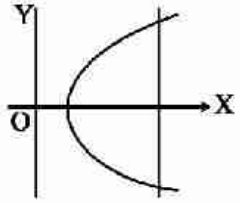
Range = For all values of x, all possible values of f(x).



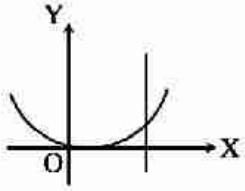
Domain = {a, b, c, d} = A
Co-domain = {p, q, r, s} = B
Range = {p, q, r}

5. Identification of a function with its graph

If we are given a graph of the relation then we can check whether the given relation is function or not . If it is possible to draw a vertical line which cuts the given curve at more than one point then given relation is not a function and when this vertical line means line parallel to Y - axis cuts the curve at only one point then it is a function.



(i)



(ii)

fig. (i) does not represents a function but (ii) represents a function.

6. Classification of Real Function

Real functions are generally classified under two categories algebric functions and transcendental functions.

(i) Algebric Function :

Algebric functions further can be classified as-

(a) Polynomial or integral Function :

The function of the form

$f(x) = a_0 x^n + a_1 x^{n-1} + ...+ a_{n-1}x + a_n$, $a_0 \neq 0$

Where $a_0, a_1, a_2, ...a_n$ are constants and $n \in N$

(b) Rational Function :

If a function $y = f(x)$ is given by $f(x) = \frac{\Phi(x)}{\Psi(x)}$

Where, $\Phi(x)$ and $\psi(x)$ are polynomial functions, then f(x) is called rational function in x

(c) Irrational Function :

The algebric function containing one or more terms having non-integral rational power of x is called irrational functions.

(ii) Transcendental Functions :

A function which is not algebraic is called a transcendental function. Trigonometric, Inverse trigonometric, Exponential, Logarithmic, etc are transcendental functions.

7. Explicit and Implicit Function

(i) Explicit Function :

A function is said to be explicit if it can be expressed directly in terms of the independent variable.

$y = f(x)$ or $x = \phi(y)$

Eg. $f(x) = x^2 \sin x$

(ii) Implicit Function :

A function is said to be implicit if it can not be expressed directly in terms of the independent variable.

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

8. Increasing and Decreasing Function

(i) Increasing Function

A function f(x) is called increasing function in the domain D if the value of the function does not decrease by increasing the value of x.

If $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

or $f'(x) > 0$ for increasing and $f'(x) \geq 0$ for not decreasing.

(ii) Decreasing Function :

A function f(x) is said to be decreasing function in the domain D if the value of the function does not increase by increasing the value of x (variable).

If $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

or $f'(x) < 0$ for decreasing and $f'(x) \leq 0$ for not increasing.

9. Even and Odd Function

(i) Even Function :

If we put (–x) in place of x in the given function and if $f(-x) = f(x)$, $\forall\ x \in$ domain then function f(x) is called even function.

(ii) Odd Function :

If we put (–x) in place of x in the given function and if $f(-x) = -f(x)$, $\forall\ x \in$ domain then f(x) is called odd function.

(iii) Properties of Even and Odd Function :

(i) The product of two even functions is even function.

(ii) The sum and difference of two even functions is even function.

(iii) The sum and difference of two odd functions is odd function.

(iv) The product of two odd functions is even function.

(v) The product of an even and an odd function is odd function.

(vi) The sum of even and odd function is neither even nor odd function.

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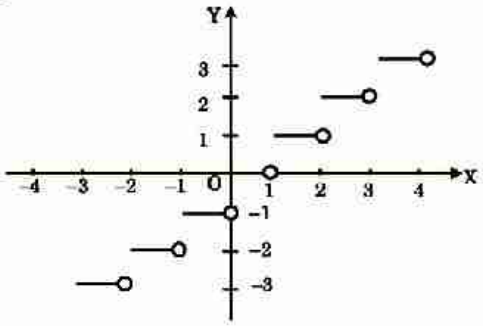
10. Greatest Integer Function

A function is said to be greatest integer function if it is of the form of f(x) = [x] where [x] = integer equal or less than x. i.e. [4.2] = 4, [– 4. 4] = – 5



Properties of G.I.F. :

(i) $[x] = x$ if x is integer

(ii) $[x + I] = [x] + I$, if I is an integer

(iii) $[x + y] \geq [x] + [y]$

(iv) If $[\phi(x)] \geq I$ then $\phi(x) \geq I$

(v) If $[\phi(x)] \leq I$ then $\phi(x) < I + 1$

(vi) If $[x] > n \Rightarrow x \geq n + 1$

(vii) If $[x] < n \Rightarrow x < n, n \in I$

(viii) $[-x] = -[x]$ if $\forall x \in I$

(ix) $[-x] = -[x] - 1$ if $x \neq$ Integer

(x) $[x + y] = [x] + [y + x - [x]] \ \forall x, y \in R$

(xi) $[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + ... + \left[x + \frac{n-1}{n}\right] = [nx]$

11. Periodic Function

A function is said to be periodic function if its each value is repeated after a definite interval. So a function f(x) will be periodic if a positive real number T exist such that, $f(x + T) = f(x)$, $\forall x \in$ Domain. Here the least positive value of T is called the period of the function.

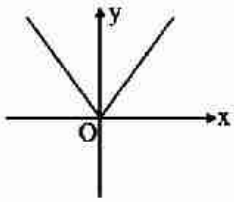
Note :

- Constant function is periodic with no fundamental period.
- If function f (x) has period T then
  (a) f(nx) has period T/n
  (b) f(x/n) has period nT
  (c) f(ax + b) has period $\frac{T}{|a|}$
- If the period of f(x) and g(x) are same (T) then the period of af(x) + bg(x) will also be T.
- If the period of f(x) is T_1 and g(x) has T_2, then the period of $f(x) \pm g(x)$ will be LCM of T_1 and T_2 provided it satisfies the definition of periodic function.
- sinx, cosx, secx and cosec x are periodic functions with period 2π
- tan x and cotx are periodic functions with period π.
- |sinx|, |cosx|, |tanx|, |cotx|, |secx| and |cosecx| are peiodic functions with period π

- $\sin^n x$, $\cos^n x$, $\sec^n x$ and $\operatorname{cosec}^n x$ are periodic functions with period 2π, when n is odd, or π when n is even.
- $\tan^n x$ and $\cot^n x$ are periodic functions with period π
- $|\sin x| + |\cos x|$, $|\tan x| + |\cot x|$ and $|\sec x| + |\operatorname{cosec} x|$ are periodic with period $\pi/2$

12. Modulus Function

It is given $n \in N$ by $y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



Properties of Modules Function :

(i) $|x| \leq a \Rightarrow -a \leq x \leq a$

(ii) $|x| \geq a \Rightarrow x \leq -a$ or $x \geq a$

(iii) $|x + y| = |x| + |y| \Rightarrow x, y \geq 0$ or $x \leq 0, y \leq 0$

(iv) $|x - y| = |x| - |y| \Rightarrow x \geq 0$ and $|x| \geq |y|$ or $x \leq 0$ and $y \leq 0$ and $|x| \geq |y|$

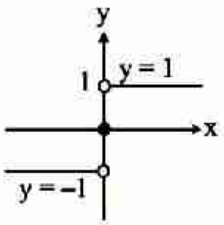
(v) $|x \pm y| \leq |x| + |y|$

(vi) $|x \pm y| \geq |x| - |y|$

13. Signum Function

The signum function f is defined as

$$\text{sgn}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$



14. Fractional Part Function

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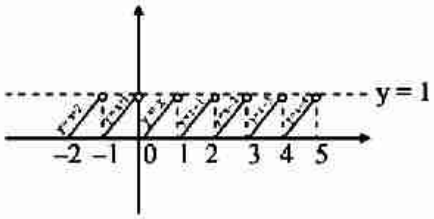
It is denoted as $f(x) = \{x\}$ and defined as

(i) $\{x\} = f$ if $x = n + f$ where $n \in I$ and $0 \leq f < 1$

(ii) $\{x\} = x - [x]$

(For proper fraction $0 < f < 1$)



15. Even & Odd Extension

(i) **Even Extension** : If a function f(x) is defined on the interval [0, a], $0 \le x \le a \Rightarrow -a \le -x \le 0$ we define f(x) in the [– a, 0] such that f(x) = f(–x). Let $I(x) = \begin{cases} f(x) & : x \in [0,a] \\ f(-x) & : x \in [-a,0] \end{cases}$

(ii) **Odd Extension :** If a function f(x) is defined on the interval [0, a], $0 \le x \le a \Rightarrow -a \le -x \le 0$

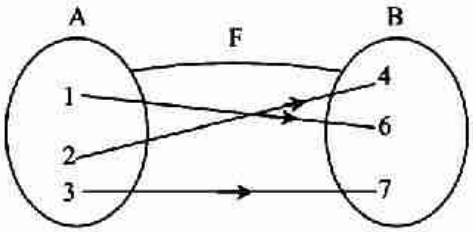
$\therefore$ $x \in [-a, 0]$, we define f(x) = – f(–x). Let be the odd extension then

$I(x) = \begin{cases} f(x), x \in [0,a] \\ -f(-x), x \in [-a,0] \end{cases}$

16. Kinds of Mapping

(i) **One-One Function :** The mapping f : A→ B is called one-one function, if different elements in A have different images in B. Such a mapping is known as injective function or an injection.

Methods to Test One-One :



(a) **Analytically :** If $x_1, x_2 \in A$, then $f(x_2) = (x_2) \Rightarrow x_1 = x_2$
or equivalently $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

(b) **Graphically :** If any line parallel to X-axis cuts the graph of the function atmost at one point, then the function is one-one.

(c) **Monotonically :** Any function which is entirely increasing or decreasing in whole domain, then f (x) is one-one.

(d) **Number of One-One Functions :** Let f : A → B be a function, such that A and B are finite sets having m and n elements respectively, (where, n > m).

The number of one-one functions

$n(n-1)(n-2)...(n-m+1) = \begin{cases} {}^nP_m \, n \ge m \\ 0, n < m \end{cases}$

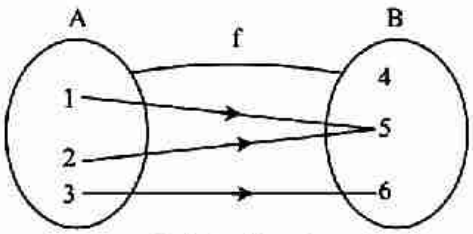
(ii) **Many-One Function :**

The function f: A ⟶ B is called many-one function, if two or more than two different elements in A have the same image in B.

Method to Test Many-one :

(a) **Analytically** If $x_1, x_2 \in A$, then

$x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$

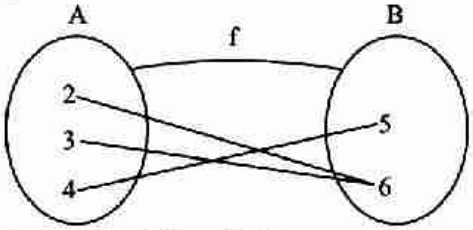


(b) **Graphically** If any line parallel to X-axis cuts the graphs of the function atleast two points, then f(x) is many-one

(c) **Monotonically** Many-one function is neither strictly increasing nor strictly decreasing. So, f(x) is not monotonic function

(d) **Number of Many-one Function** Let f: A → B be a function such that A and B are finite sets having m and n elements respectively.
The number of many-one function = n^m

(iii) **Onto** (Surjective) **Function :**
If the function f:A → B is such that each element in B (co-domain) is the image of atleast one element of A then we say that f is function of A onto B. Thus. f: A → B, such that f(A) = B,

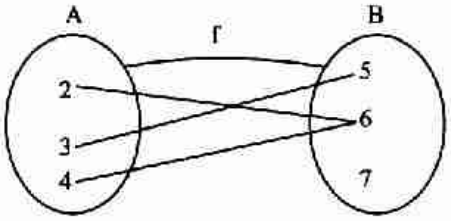


Note : Every polynomial function f : R → R of degree odd is onto
Number of Onto (surjective) **Functions** Let A and B are finite sets having m and n elements respectively, such that $1 \le n \ge m$, then number of onto (surjective) functions from A to B is

$\sum_{r=1}^{n}(-1)^{2-m}C_r r^m$ Coefficient of x^2 in $n!\,(e^x-1)^r$

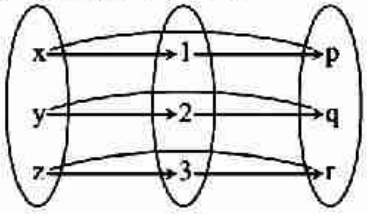
Note A one-one and onto function is called bijective function.

(iv) **Into Function :**
If f : A → B is such that there exists atleast one element in co-domain which is not the image of any element in domain, then f(x) is into.
Thus, f : A → B, such that f(x) ⊂ B
i.e. Range ⊂ Co-domain



17. Composite Function

If f : A → B and g : B → C are two function then the composite function of f and g, gof A → C will be defined as gof (x) = g [f(x)], ∀ x ∈ A.



Properties of Composite Function :

(i) If f and g are two functions then for composite of two functions fog ≠ gof.

(ii) Composite functions obeys the property of associativity i.e. fo(goh) = (fog)oh.

(iii) Composite function of two one-one onto functions if exist, will also be a one-one onto function.

18. Inverse Function

If $f : A \to B$ be a one-one onto (bijection) function, then the mapping $f^{-1} : B \to A$ which associates each element $b \in B$ with element $a \in A$, such that $f(a) = b$, is called the inverse function of the function $f : A \to B$

$f^{-1} : B \to A$, $f^{-1}(b) = a \Rightarrow f(a) = b$

(i) Properties: For the existence of inverse function, it should be one-one and onto.

(a) Inverse of a bijection is also a bijection function.

(b) Inverse of a bijection is unique.

(c) $(f^{-1})^{-1} = f$

(d) If f and g are two bijections such that (gof) exists then $(gof)^{-1} = f^{-1}og^{-1}$

(e) If $f : A \to B$ is a bijection then $f^{-1} : B \to A$ is an inverse function of f.

$f^{-1}of = I_A$ and $fof^{-1} = I_B$.

(ii) Algebra of Function :

(a) $(fog)(x) = f[g(x)]$

(b) $(fof)(x) = f[f(x)]$

(c) $(gog)(x) = g[g(x)]$

(d) $(fg)(x) = f(x) \cdot g(x)$

(e) $(f \pm g)(x) = f(x) \pm g(x)$

(f) $(f/g)(x) = \dfrac{f(x)}{g(x)}$, $g(x) \neq 0$

* Composite functions is not commutative

* Let f and g are two functions then if f & g are injective or surjective or bijective then "gof " also injective or surjective or bijective.

19. Important points to be remembered

If x, y are Independent Variables then :

(i) If $f(xy) = f(x) + f(y) \Rightarrow f(x) = k \log x$

(ii) If $f(xy) = f(x) \cdot f(y) \Rightarrow f(x) = x^n$, $n \in R$

(iii) If $f(x + y) = f(x).f(y) \Rightarrow f(x) = a^{kx}$

(iv) If $f(x + y) = f(x) + f(y) \Rightarrow f(x) = x$

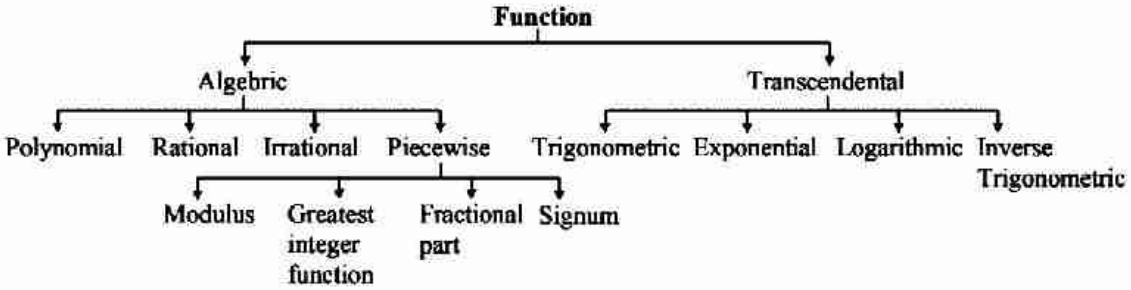
(v) If $f(x + y) = f(x) = f(y) \Rightarrow f(x) = k$, here k is constant

(vi) By considering a general nth degree polynomial and writing the expression

$$f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) \Rightarrow f(x) = \pm x^n + 1$$

20. Some Functions & Their Graphs

Classification of Function :



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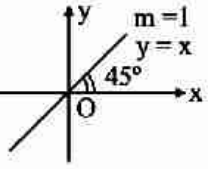
Graphs of Function :

(i) Polynomials

(a) $y = f(x) = mx$, where m is a constant$(m \neq 0)$

(1)



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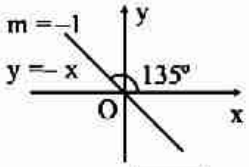

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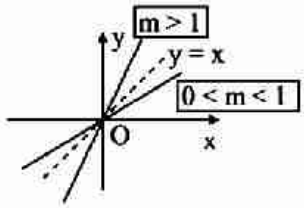

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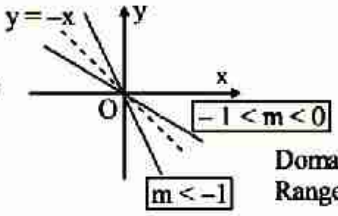
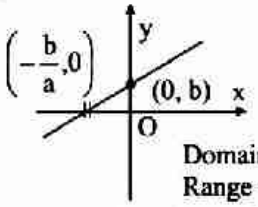


Domain = R
Range = R

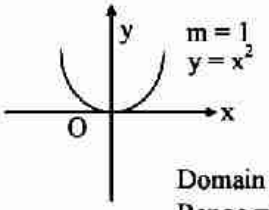
$y = mx$ is continuous function & one-one function $(m \neq 0)$

(5) $y = ax + b$, where a & b are constant

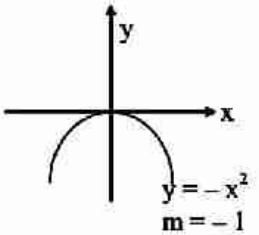


Domain = R
Range = R

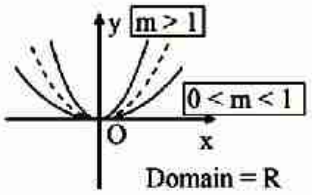
(b) $y = mx^2$, $(m \neq 0)$, $x \in R$



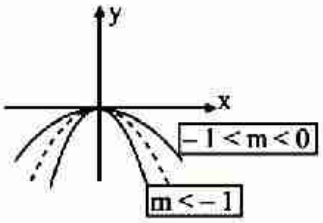
Domain = R
Range = $[0, \infty]$



Domain = R
Range = $(\infty, 0]$



Domain = R
Range = [0, ∞)

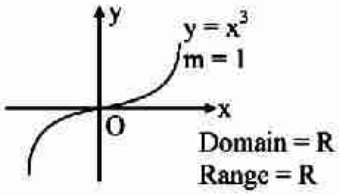


Domain = R
Range = [0, ∞)

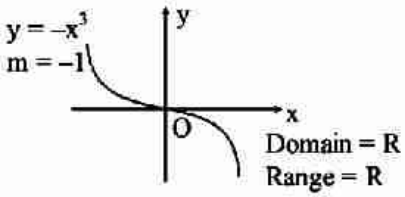
$y = mx^2$ is continuous, even function and many-one function.

Graph is symmetric about y-axis

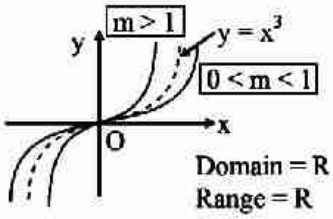
(c) $y = mx^3, (m \neq 0), x \in R$



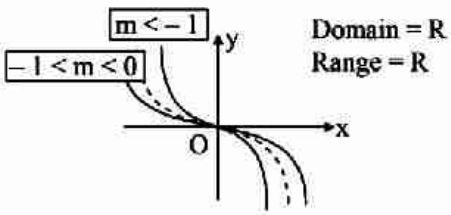
Domain = R
Range = R



Domain = R
Range = R



Domain = R
Range = R



Domain = R
Range = R

$y = mx^3$ is continuous, odd one-one & increasing function.

Graph is symmetric about origin.

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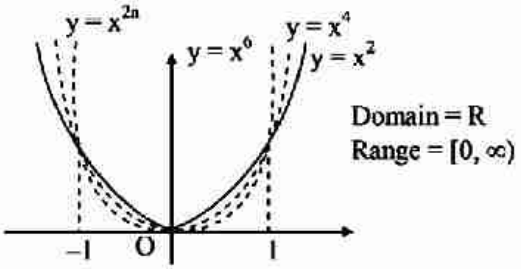
(d) $y = x^{2n}, n \in N$



$f(x) = x^{2n}$ is an even, continuous function. It is many one function.

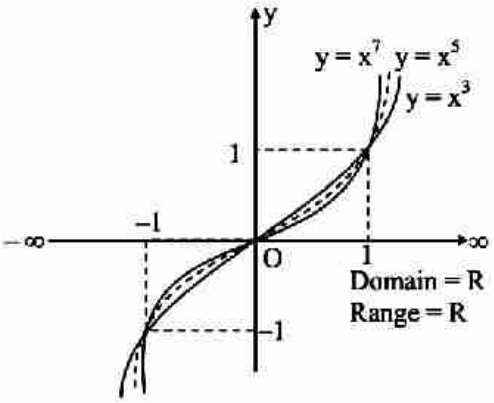
From the Graph :

For $|x| \le 1 \Rightarrow x^6 \le x^4 \le x^2$

For $|x| > 1 \Rightarrow x^6 > x^4 > x^2$

Graph is symmetric about y-axis

(e) $y = x^{2n+1}, n \in N$



$f(x) = x^{2n+1}$, $n \in N$ is an odd continuous function. It is always one-one function.

From the Graph :

For $x \in (-\infty, -1] \cup [0, 1]$

$\Rightarrow \quad x^8 \ge x^6 \ge x^7$

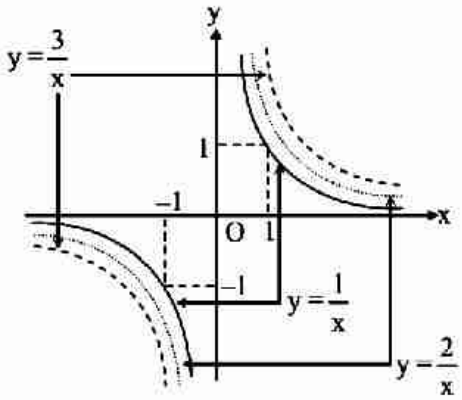
For $x \in (-1, 0) \cup (1, \infty)$

$\Rightarrow \quad x^8 < x^6 < x^7$

Graph is symmetric about origin.

(ii) Rational Function :

(a) $y = \frac{m}{x}, x \ne 0, m \ne 0$



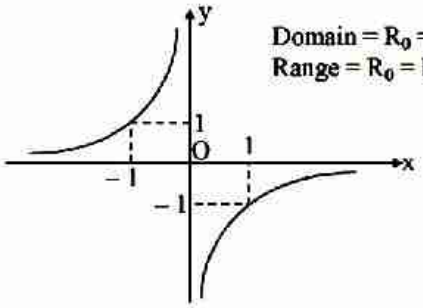
$y = \frac{m}{x}$ is odd function discontinuous at $x = 0$.

Domain = $R_0 = R - \{0\}$

Range = $R_0 = R - \{0\}$

This type of graph $xy = m$ is called rectangular hyperbola.

(b) $y = -\frac{1}{x}, x \neq 0$



Domain = $R_0 = R - \{0\}$
Range = $R_0 = R - \{0\}$

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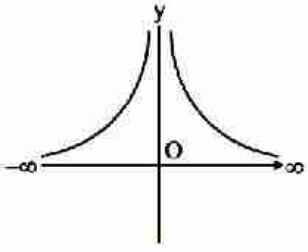
$y = -\frac{1}{x}$ is an odd & one-one function discontinuous at $x = 0$

(c) $y = \frac{1}{x^2}, x \neq 0$

$y = \frac{1}{x^2}$ is even function, many one and discontinuous at $x = 0$

Domain = R_0

Range = $R^+ = (0, \infty)$



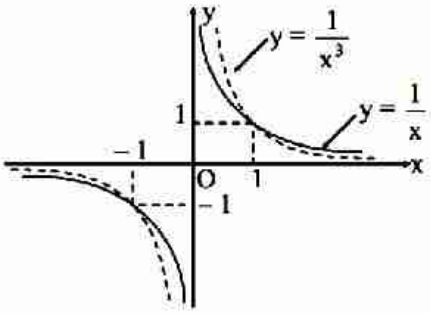
(d) $y = \frac{1}{x^{2n-1}}, n \in N$

An odd one-one function discontinuous at $x = 0$

From Graph :

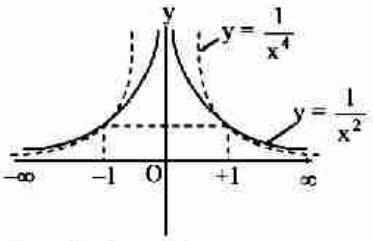
$0 < x < 1 \Rightarrow \frac{1}{x^3} > \frac{1}{x}$

$x > 1 \Rightarrow \frac{1}{x^3} < \frac{1}{x}$.



(e) $y = \frac{1}{x^{2n}}$, $n \in N$

$y = \frac{1}{x^{2n}}$ is an even function & many one, discontinuous at $x = 0$.

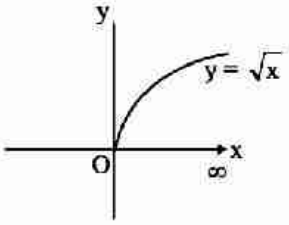


Domain = R_0
Range = R^+

(iii) Irrational Function

(a) $y = \sqrt{x} = x^{1/2}$

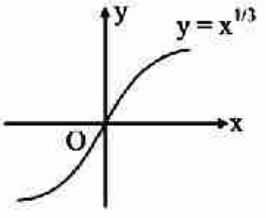
This function is neither even nor odd, one-one and continuous function in it's domain. It is an increasing function.



Domain = $[0, \infty)$
Range = $[0, \infty)$

(b) $y = \sqrt[3]{x} = x^{1/3}$

This function is an odd, continuous and one-one function. It is an increasing function.



Domain = R
Range = R

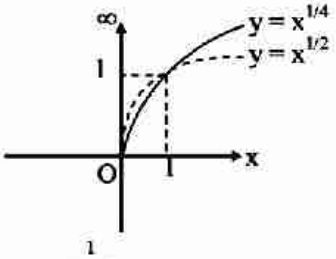
(c) $y = x^{1/2n}$, $n \in N$

This function is neither even nor odd, continuous and increasing function it is also one-one function

From graph

$0 < x < 1 \Rightarrow > \sqrt[2]{x} > \sqrt[4]{x}$

$x > 1 \Rightarrow x^{1/2} < x^{1/4}$



Domain = $R^+ = [0, \infty)$
Range = $R^+ = [0, \infty)$

(d) $y = x^{2n+1}$, $n \in N$

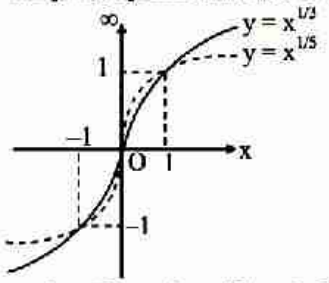
This function is an odd, continuous, one-one and increasing function

From Graph :

$x \in (-\infty, -1) \cup (0, 1)$, $x^{1/3} > x^{1/5}$

$x \in (-1, 0) \cup (1, \infty)$, $x^{1/3} < x^{1/5}$

Graph is symmetric about origin.



Domain = R
Range = R

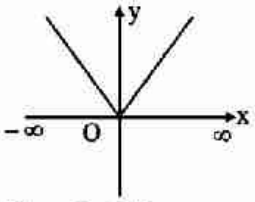
(iv) Piecewise Function (Special Function)

(a) Modulus Function

$$y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

This is an even, continuous, many one function.

Graph is symmetric about y-axis

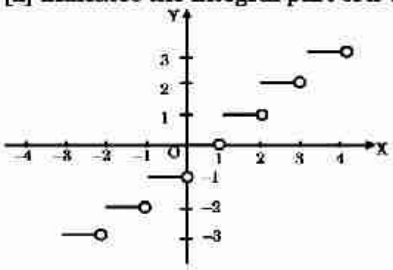


Domain = R

Range = $[0, \infty)$

(b) Greatest Integer Function

[x] indicates the integral part of x which is nearest and smaller to x. It is also called step function.



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$$y = [x] = \begin{cases} -1, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ +1, & 1 \le x < 2 \\ +2, & 2 \le x < 3 \\ +3, & 3 \le x < 4 \end{cases}$$

$\Rightarrow \quad y = [x] = k,\ k \le x < k+1,\ k \in I$

Important Result from graph

$x - 1 < [x] \le x$

(c) Fractional Part of x

$y = \{x\}$

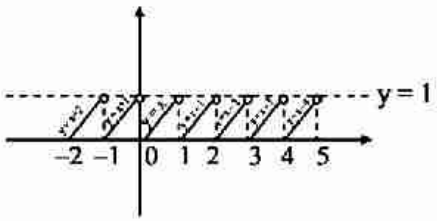
Any number is sum of integral & fractional part i.e. $x = [x] + \{x\}$

$\therefore\ y = \{x\} = x - [x] = x - k,\ k \le x < k+1$

$$y = \{x\} = \begin{cases} x+1, & ..... \\ x, & 0 \le x < 1 \\ x-1, & 1 \le x < 2 \\ x-2, & 2 \le x < 3 \\ ....., & ..... \end{cases}$$

Domain = R

Range = [0, 1)

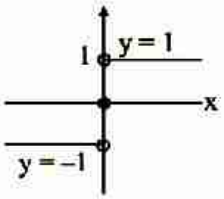


From graph $0 \le \{x\} < 1$

This function is a periodic function with period 1. This is also many-one function discontinuous at $x \in I$.

(d) $y = \text{sgn}(x) = \begin{cases} \dfrac{|x|}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases} = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$

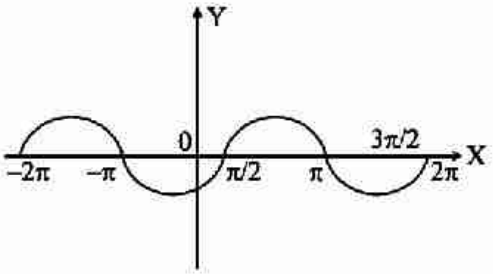
This is odd, many-one function discontinuous at x = 0



Domain = R
Range = {–1, 0, 1}

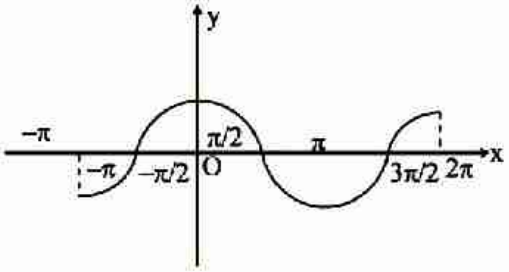
(v) Trigonometrical Function :

(a) $f(x) = \sin x$.



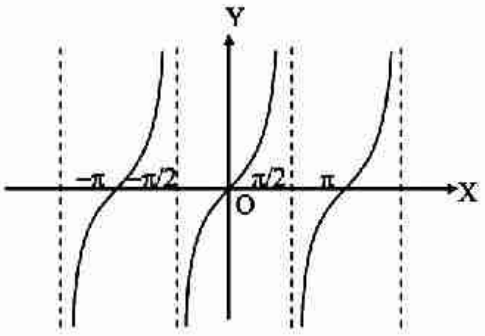
It is a continuous and many one function with period 2π.

(b) $f(x) = \cos x$



It is a continuous and many-one function having a period $2.\pi$

(c) $f(x) = \tan x$



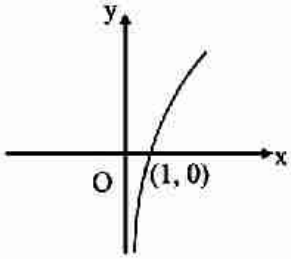
Domain = $R - (2n+1)\,\pi/2,\ n \in I$

Range = R

It is a discontinuous function at $x = (2n+1)\pi/2$, $n \in I$ and periodic with period π.

(vi) Logarithmic Function :

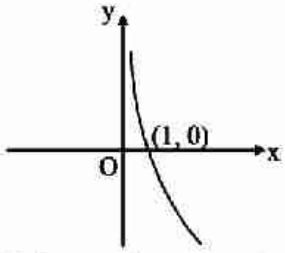
(a) $f(x) = \log_a x \quad (a > 1)$



Domain = R^+
Range = R

It is a continuous and one-one function.

(b) $f(x) = \log_a x \quad (a<1)$

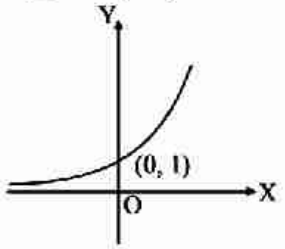


Domain = R^+
Range = R

It is a continuous and one-one function.

(vii) Exponential Function :

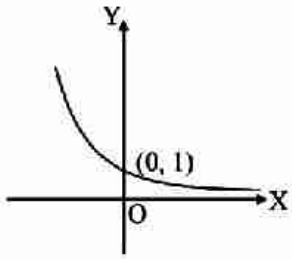
(a) $f(x) = a^x \quad (a>1)$



Domain = R
Range = R^+

It is a continuous and one-one function.

(b) $f(x) = a^x \qquad (a<1)$

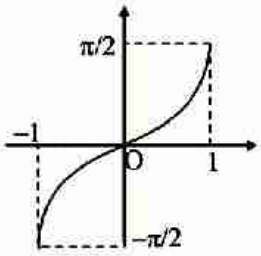


Domain = R
Range = R^+

It is a continuous and one-one function.

(viii) Inverse Trigonometric Function :

$y = \sin^{-1} x$

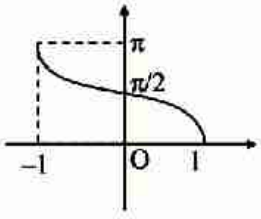


Domain = [−1, 1]

Range = $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Odd function

$y = \cos^{-1} x$



Domain = [−1, 1]
Range = [0, π]
Neither even nor odd

$y = \tan^{-1} x$

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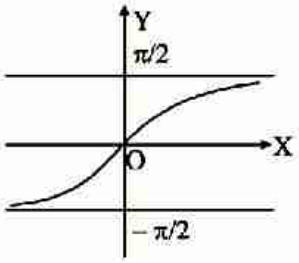
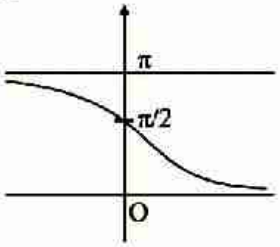


Domain = R

Range = $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Odd function

$y = \cot^{-1} x$

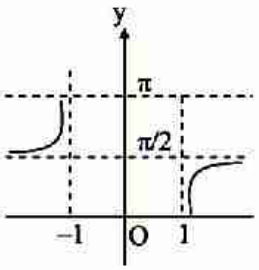


Domain = R

Range = $(0, \pi)$

Neither even nor odd

$y = \sec^{-1} x$

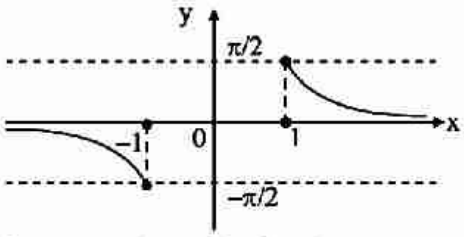


Domain = $(-\infty, -1] \cup [1, \infty)$

Range = $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

Neither even nor odd

$y = \operatorname{cosec}^{-1} x$



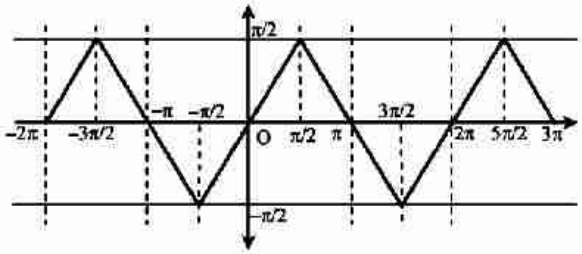
Domain = $(-\infty, -1] \cup [1, \infty)$

Range = $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

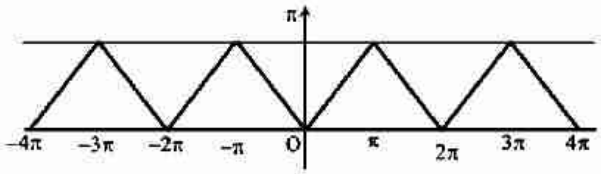
Odd function

All inverse trigonometric functions are monotonic.

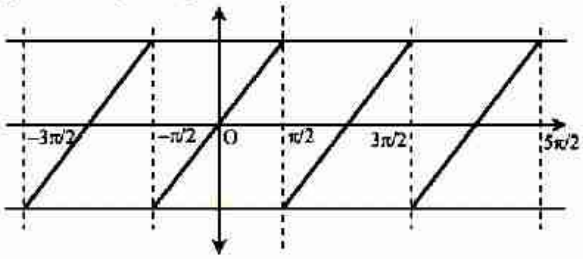
(ix) (a) **$y = \sin^{-1}(\sin x)$:**



(b) **$y = \cos^{-1}(\cos x)$:**



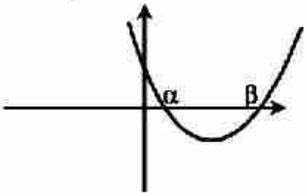
(c) $y = \tan^{-1}(\tan x)$



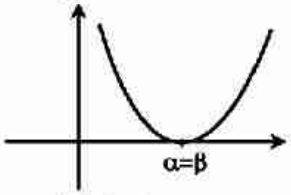
(x) Graph of Quadratic Function

$f(x) = ax^2 + bx + c$, $D = b^2 - 4ac$

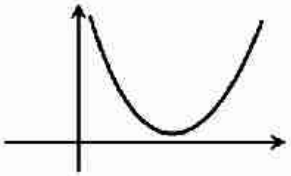
(a) $a > 0$; $D > 0$



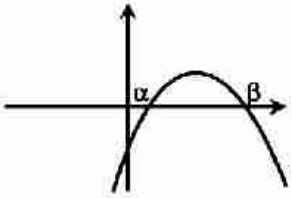
(b) $a > 0$; $D = 0$



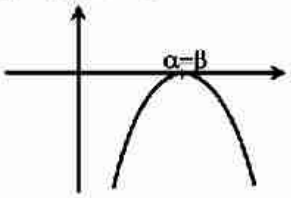
(c) $a > 0$; $D < 0$



(d) $a < 0$; $D > 0$



(e) $a < 0$; $D = 0$



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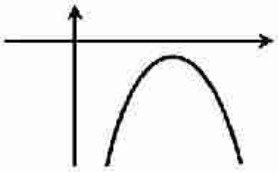
(f) $a < 0;\ D < 0$



(xi) Plotting of Algebraic Curve

From : $f(x) = (x-\alpha)^a (x-\beta)^b$ ....., $a, b \in N$.

Method :

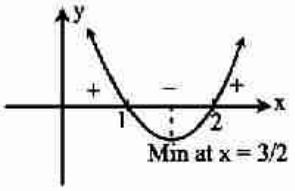
(a) Put $f(x) = 0$ to get change points.

(b) Write these points on the number line.

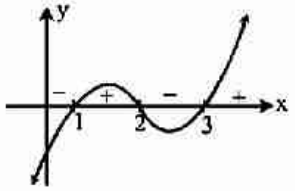
(c) Write sign scheme of wavy curve method.

(d) Wherever positive sign is occurring, the graph is above x-axis and wherever the sign is negative the graph is below x-axis.

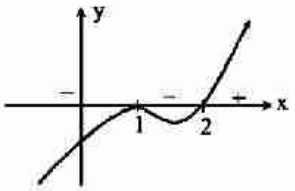
Ex. $y = (x-1)(x-2)$



Ex. $y = (x-1)(x-2)(x-3)$



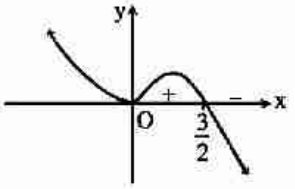
Ex. $y = (x-1)^2 (x-2)$



Ex. $y = 3x^2 - 2x^3$

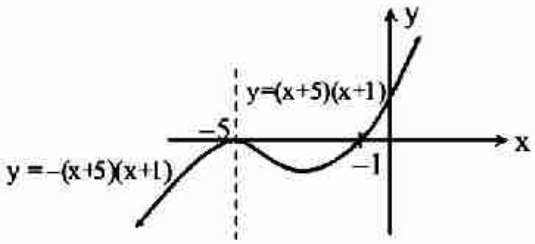
$\Rightarrow\ y = x^2(3-2x)$

$\Rightarrow\ y = -x^2(2x-3)$



Ex. $y = |x+5|\,(x+1)$

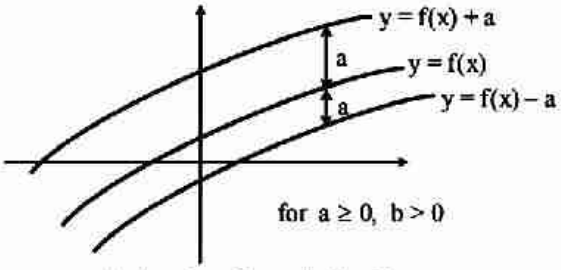
$$\Rightarrow\ y = \begin{cases} (x+5)(x+1) & ;\ x \geq -5 \\ -(x+5)(x+1) & ;\ x < -5 \end{cases}$$



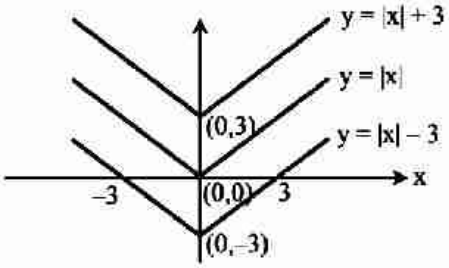
(xii) To draw y = f(x) ± a from y = f(x)

For f(x) + a, shift f(x) upward by 'a' units.

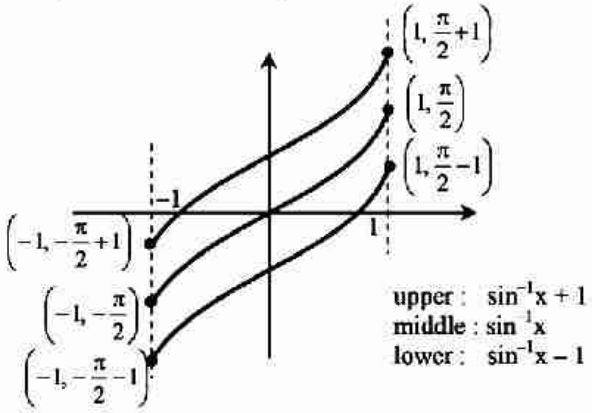
For f(x) – a, shift f(x) downward by 'a' units.



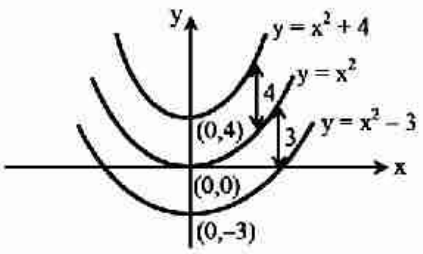
Ex. y = |x| + 3 and y = |x| – 3



Ex. $y = \sin^{-1}x + 1$ and $y = \sin^{-1}x - 1$



Ex. $y = x^2 + 4$ & $y = x^2 - 3$



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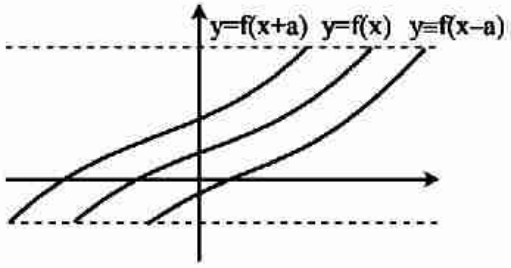
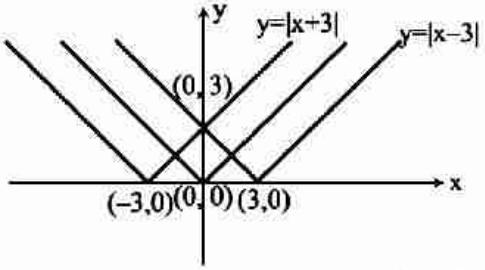
(xiii) To draw y = f(x ± a) from y = f(x)

For f(x + a) : shift y = f(x) left by 'a' units

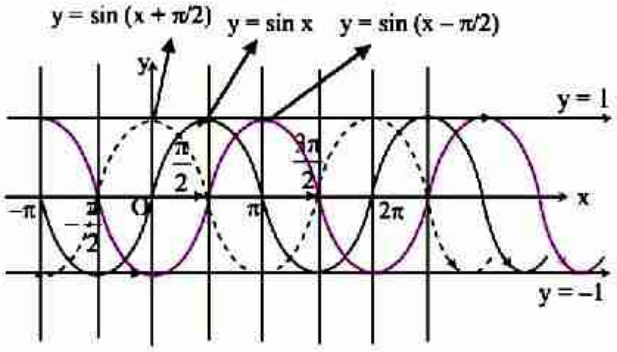
For f(x – a) : shift y = f(x) right by 'a' units



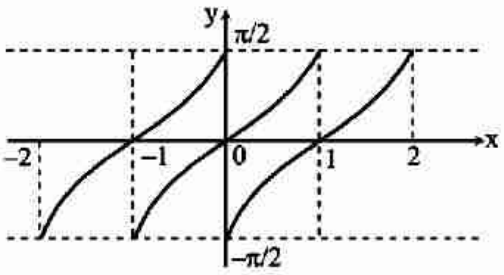
Ex. y = |x – 3| and y = |x + 3|



Ex. $y = \sin\left(x - \frac{\pi}{2}\right)$ and $y = \sin\left(x + \frac{\pi}{2}\right)$



Ex. y = sin ¹(x –1) and y = sin ¹(x + 1)



Left : $\sin^{-1}(x + 1)$

Middle : $\sin^{-1}x$

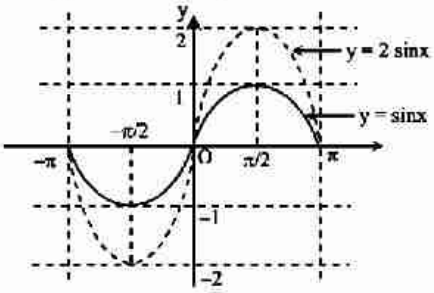
Right : $\sin^{-1}(x - 1)$

(xiv) To draw y = af(x) from y = f(x)

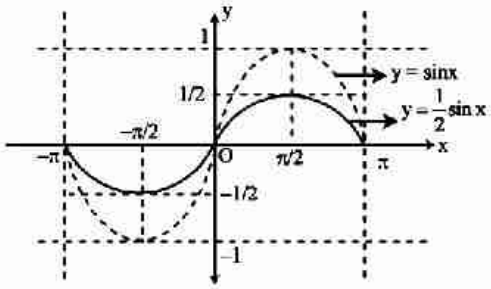
If a > 1, expand graph 'a' times along y-axis

If $a = \frac{1}{k}$, $k > 1$, contract graph 'k' times along y-axis

Ex. $y = \sin x$ and $y = 2\sin x$



Ex. $y = \sin x$ and $y = \frac{1}{2}\sin x$

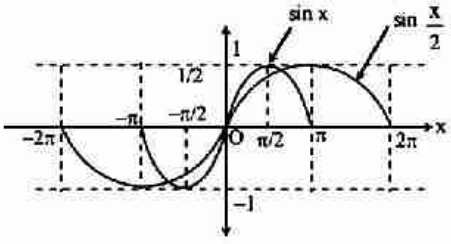


(xv) To draw y = f(ax) from y = f(x)

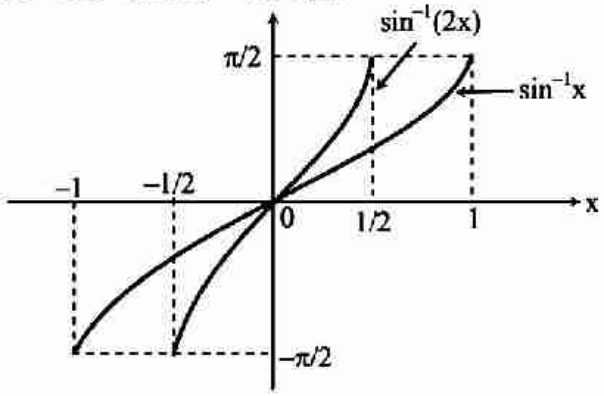
For $a > 1$, contract the graph of f(x) by 'a' times along x-axis.

For $a = \frac{1}{k}$, $k > 1$, expand the graph of f(x) by 'k' times along x-axis.

Ex. $y = \sin x$ and $y = \sin \frac{x}{2}$



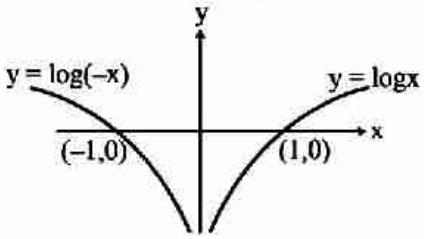
Ex. $y = \sin^{-1} x$ and $y = \sin^{-1}(2x)$



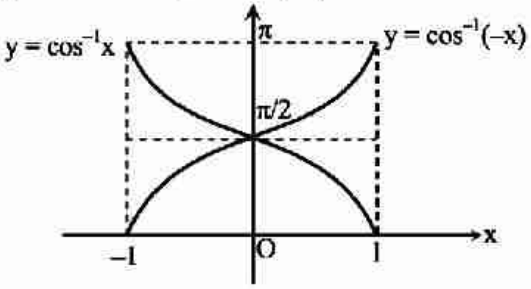
(xvi) To draw y = f(–x) from y = f(x)

Take mirror image of y = f(x) in y-axis

Ex. y = log x and y = log (–x)



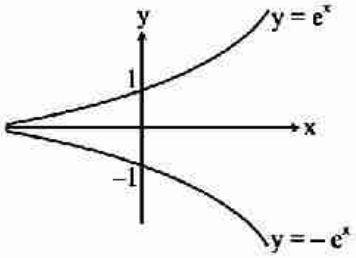
Ex. $y = \cos^{-1}x$ and $y = \cos^{-1}(-x)$



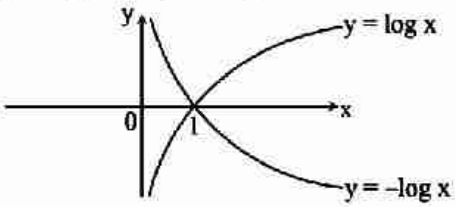
(xvii) To draw y = –f(x) from y = f(x)

Take mirror image of y = f(x) in the x-axis

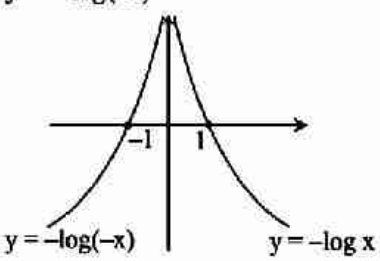
Ex. $y = e^x$ and $y = -e^x$



Ex. y = log x and y = –log x



Ex. y = – log(–x)



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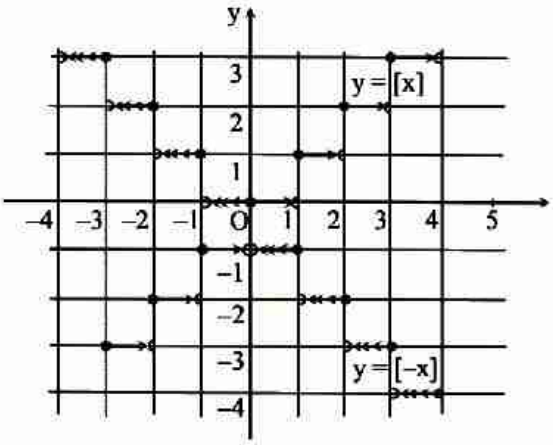
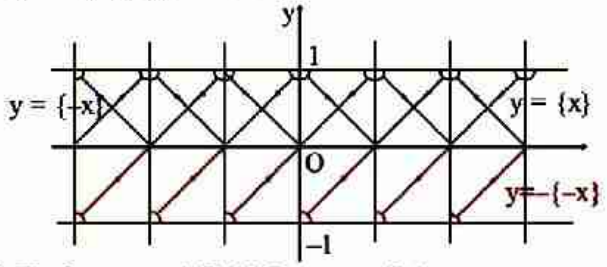
Ex. $y = [x],\ y = [-x]$



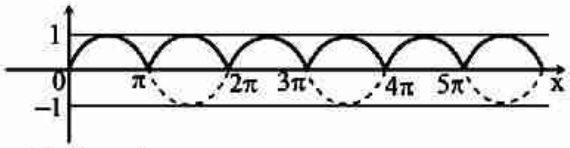
Ex. $y = -\{-x\}$, ({.} is F.P.F)



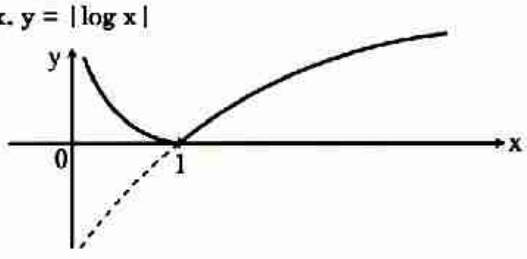
(xviii) To draw $y = |f(x)|$ from $y = f(x)$

1. Draw graph of f(x)
2. Take mirror image in x-axis for negative portion
3. Erase negative portion

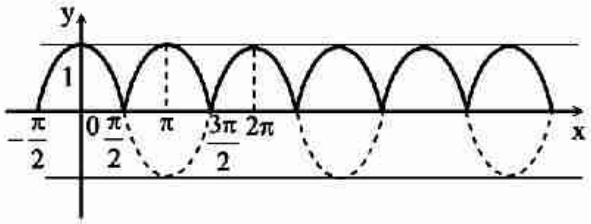
Ex. $y = |\sin x|$



Ex. $y = |\log x|$

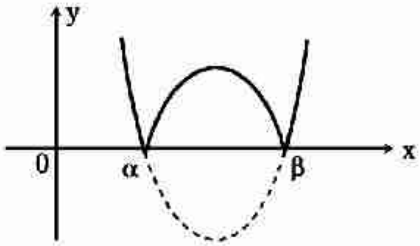


Ex. $y = |\cos x|$

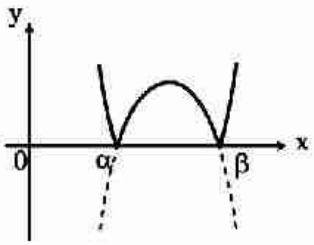


Ex. $y = |ax^2 + bx + c|$

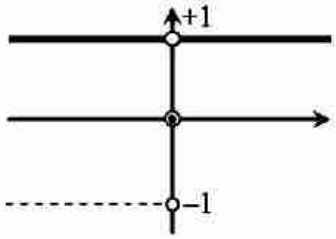
for $a > 0 ; D > 0$



for $a < 0; D > 0$



Ex. $y = |\ \text{sgn}\ (x)\ |$



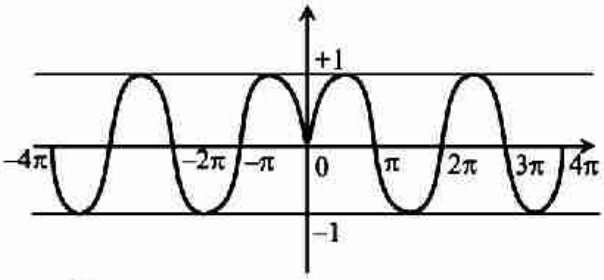
(xix) To draw :

$y = f(|x|)$ from $y = f(x)$

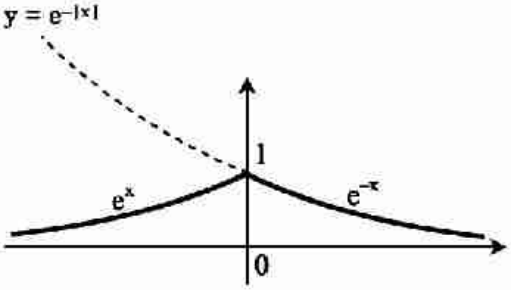
(a) Draw graph for positive values of x.

(b) Take mirror image in y-axis.

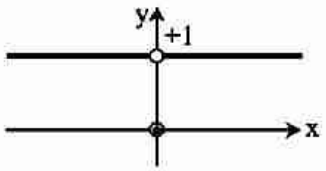
Ex. $y = \sin |x|$



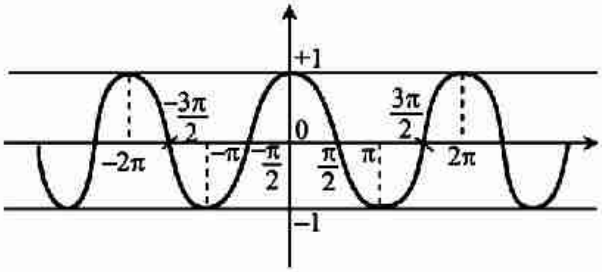
Ex. $y = e^{-|x|}$



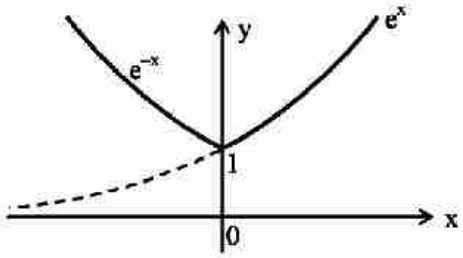
Ex. $y = \text{sgn} \mid x \mid$



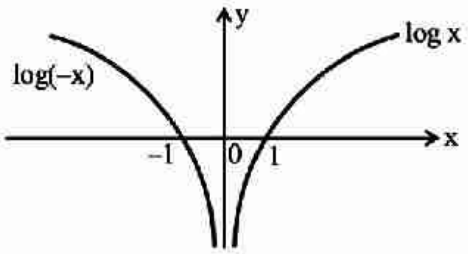
Ex. $y = \cos \mid x \mid = \cos x$



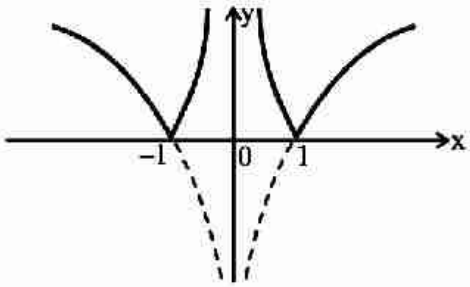
Ex. $y = e^{|x|}$



Ex. $y = \log \mid x \mid$



Ex. $y = |\log \mid x \mid|$



(xx) To draw $\mid y \mid = f(x)$ from $y = f(x)$

(a) Draw graph of f(x)

(b) Eliminate negative portion

(c) Take mirror image in x-axis for positive portion

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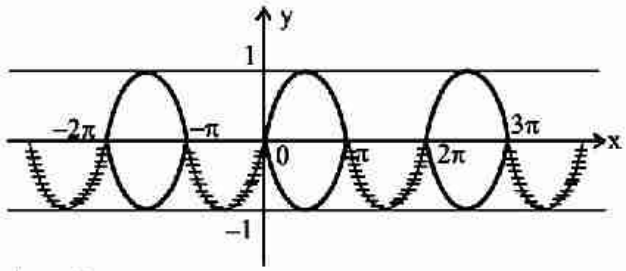
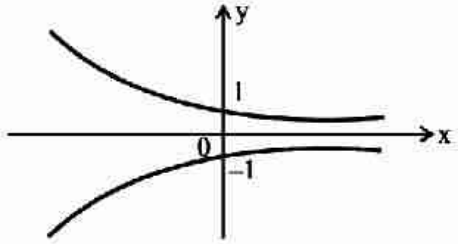
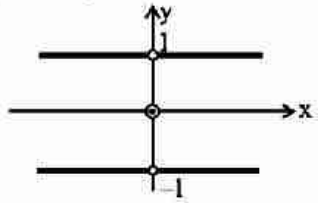
Ex. $|y| = \sin x$



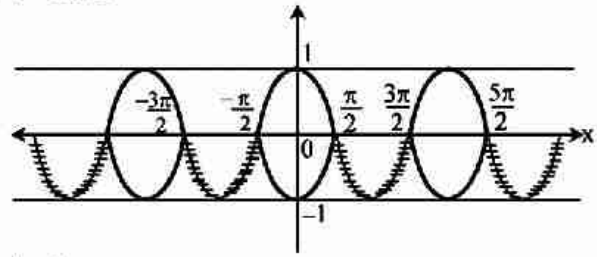
Ex. $|y| = e^{-x}$



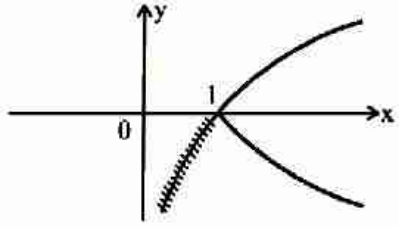
Ex. $|y| = |\text{sgn}\,|x||$



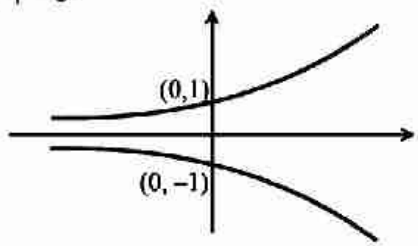
Ex. $|y| = \cos x$



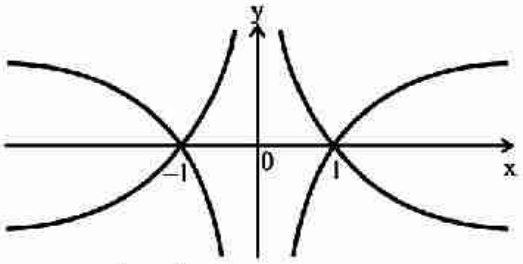
Ex. $|y| = \log x$



Ex. $|y| = e^{x}$



Ex. $|y| = |\log|x||$



(xxi) To trace $y = [f(x)]$ from $y = f(x)$ (where [.] is GIF)

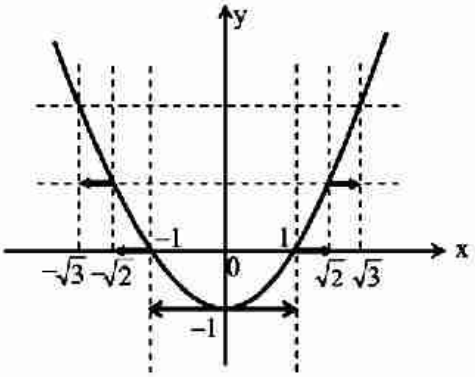
(a) Plot $y = f(x)$

(b) Mark the interval of unit length with integers as end points on y-axis.

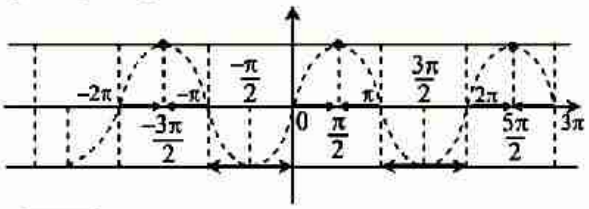
(c) Mark the corresponding intervals on the x-axis.

(d) Plot the value of [f(x)] for each intervals.

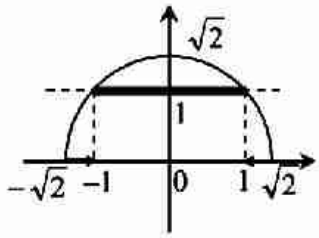
Ex. $y = [x^2 - 1]$ and $y = (x^2 - 1)$.



Ex. $y = [\sin x]$ and $y = \sin x$.



Ex. $y = [\sqrt{2 - x^2}]$



(xxii) To draw $y = f[x]$ from $y = f(x)$ (where [.] is GIF)

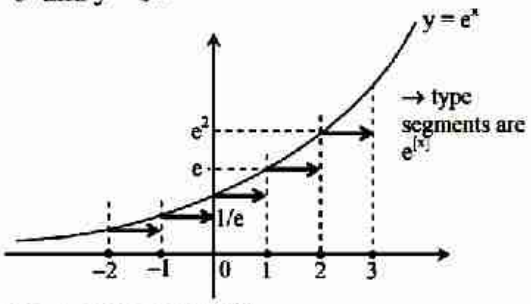
(a) Plot lines parallel to y-axis for integral values of x.

(b) Mark the points at integers on the curve

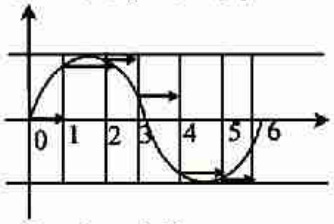
(c) Take lower marked point of x say if

$n < x < n + 1$, then take the point at $x = n$ and draw a horizontal line to nearest formed by $x = n + 1$

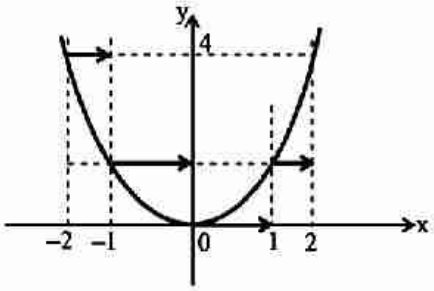
Ex. $y = e^x$ and $y = e^{[x]}$



Ex. $y = \sin x$ and $y = \sin [x]$



Ex. $y = x^2$ and $y = [x]^2$

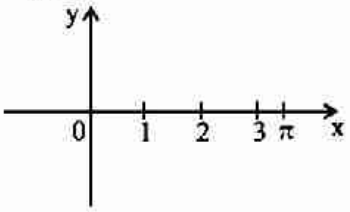


Ex. $y = [\sin[x]]$; $0 \le x \le \pi$

Look at the graph of sin[x], for $0 \le x \le \pi$.

All the segments are below 1 so $y = [\sin[x]] = 0$

Thus

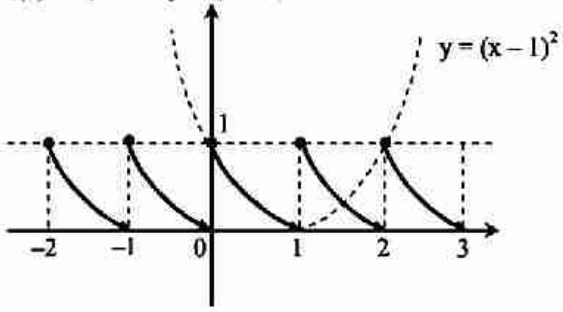


(xxiii) To draw $y = f(\{x\})$ from $y = f(x)$

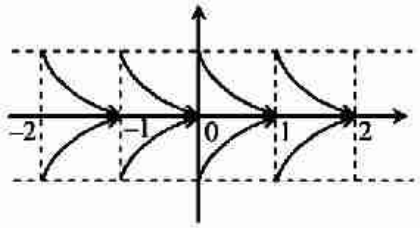
Retain the graph of f(x) for values of $x \in [0, 1]$.

It should be repeated from rest of unit intervals by looking periodicity 1.

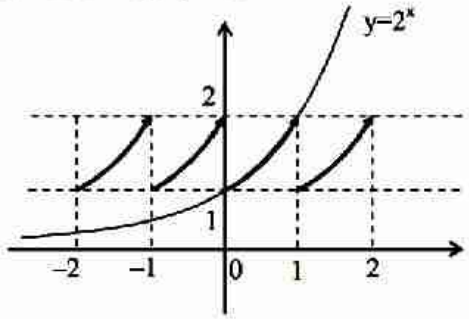
Ex. $y = (\{x\} - 1)^2$ and $y = (x - 1)^2$



Ex. $|y| = (\{x\} - 1)^2$



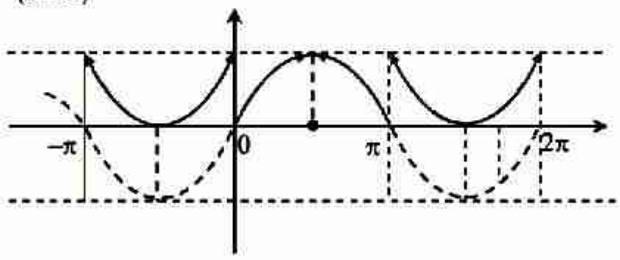
Ex. $y = 2^x/2^{[x]}$ i.e. $y = 2^{\{x\}}$



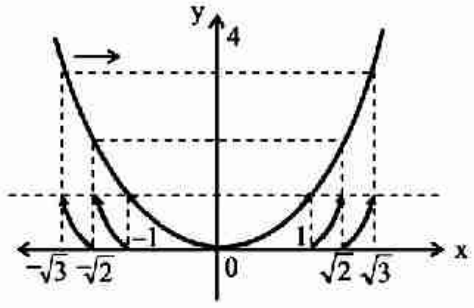
(xxiv) To draw $y = \{f(x)\}$ from $y = f(x)$

Plot horizontal lines for all integral values of y and for the point of intersection on $y = f(x)$, plot vertical lines and translate the graph for $y = 0$ and $y = 1$

Ex. $y = \{\sin x\}$



Ex. $y = \{x^2\}$

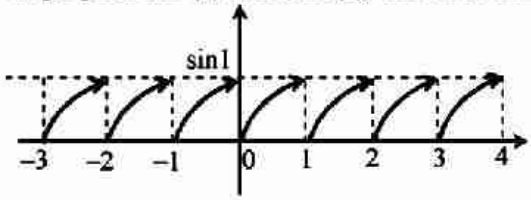


Ex. $y = \{\sin \{x\}\}$

As $\sin \{x\}$ is between $[0, \sin 1)$

$\because$ $\sin 1 < 1$

So graph of $\sin \{x\}$ and $\{\sin \{x\}\}$ both are same.



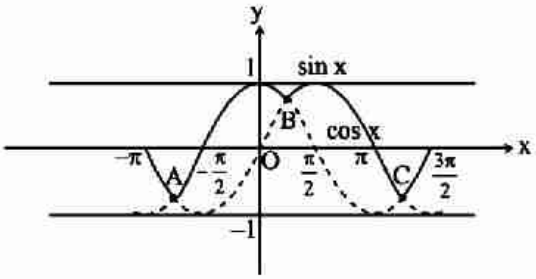
(xxv) To draw Max. or Min. {f(x).g(x)}

First draw both the curves f(x) and g(x). Observe their points of intersection. Among these points which one is above and which one is below.

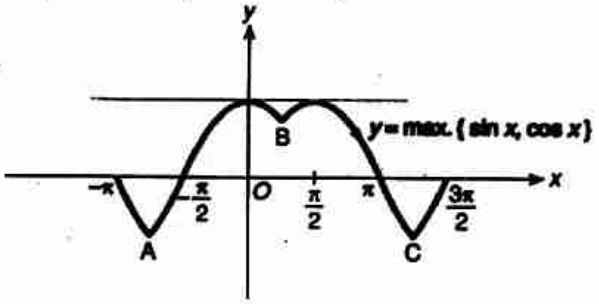
For max. take upper portions and for min take lower portions.

Ex. Sketch the graph of

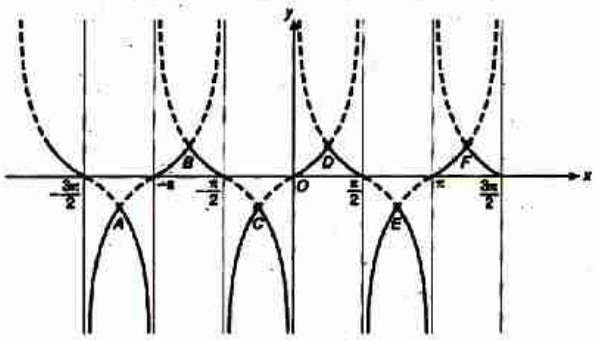
$y = \max.\{\sin x, \cos x\}, \forall\, x \in \left(-\pi, \frac{3\pi}{2}\right)$



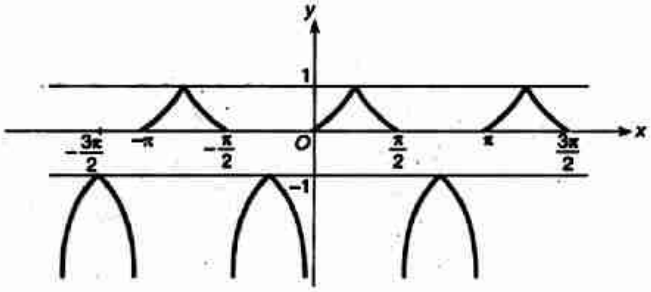
∴ **Graph of max {sin x, cos x}**



Ex. Sketch the graph for y = min{tan x, cot x}



Graph of min {tan x, cot x}



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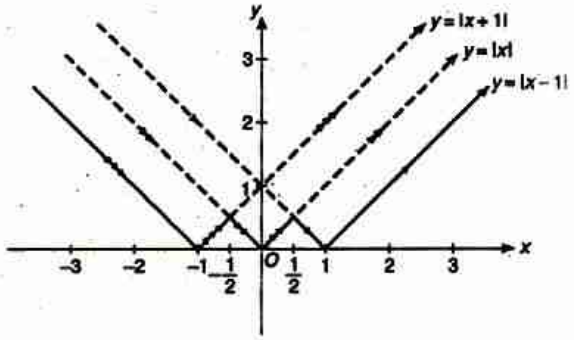
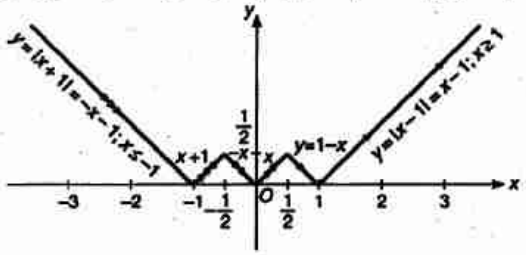
Ex. Sketch the curve $y = \min\{|x|, |x-1|, |x+1|\}$

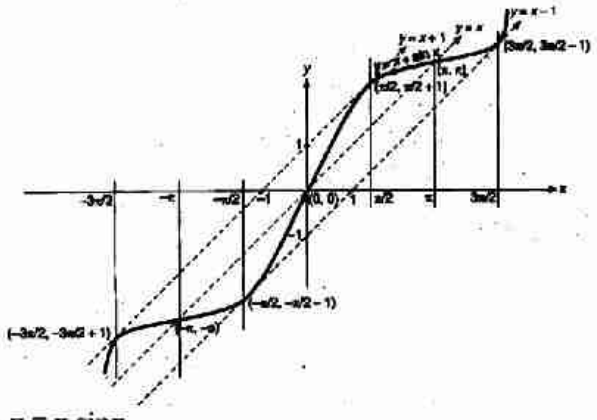


Graph for $y = |x|, y = |x-1|, y = |x+1|$

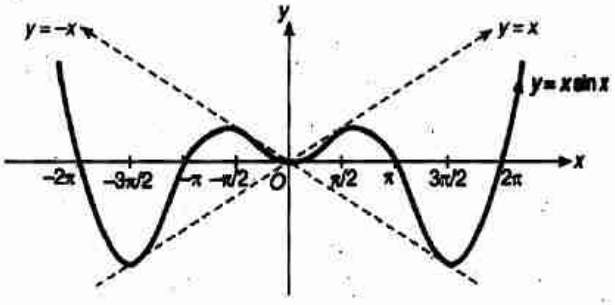


(xxvi) To draw $y = x + \sin x$ and $y = x.\sin x$

Ex. $y = x + \sin x$



Ex. $y = x.\sin x$

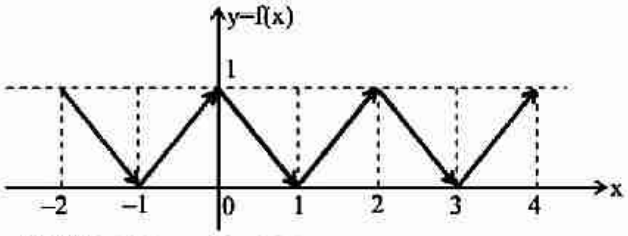


(xxvii) To draw

$$f(x) = \begin{cases} x-[x] & \text{if } [x] \text{ is odd} \\ 1+[x]-x & \text{if } [x] \text{ is even} \end{cases}$$

i.e.

$$f(x) = \begin{cases} \{x\} & \text{if } [x] = \pm 1, \pm 3.... \\ 1-\{x\} & \text{if } [x] = 0, \pm 2, \pm 4..... \end{cases}$$



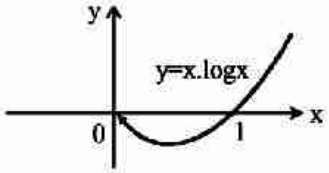
(xxviii) To draw y = x log x

(a) Take $x > 0$

(b) $\lim_{x \to 0} x.\log x \to 0$

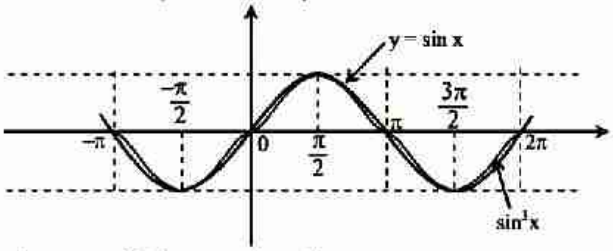
(c) $x = 1, y = 0$

(d) $y = x \log x$ is continuous + differentiable for $x > 0$

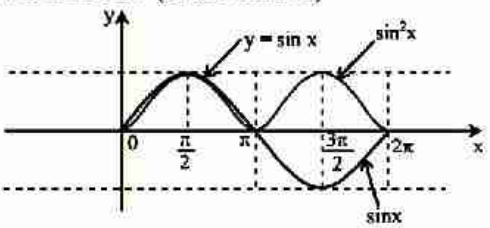


(xxix) Graphs of $T^n(x)$, $n \in N$, like $\sin^2 x$, $\sin^3 x$ etc.

$\sin^n x$. $n \in N$ (odd natural)

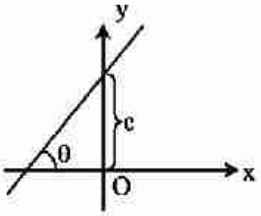


$\sin^n x$. $n \in N$ (even natural)

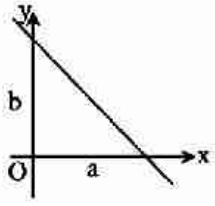


(xxx) Straight Line :

(a) Slope and y-Intercept Form :



(b) Double Intercept Form : $\frac{x}{a} + \frac{y}{b} = 1$



SHORT TRICKS

Trick -1 Let $y = f(x)$ is a function and it satisfies the relation
$f(x + a) + f(x + b) =$ constant
then period of this function is $2|b - a|$

Q.1 If $f(x) + f(x + 7) = 10$, then the period of $f(x)$ is-
(A) 7 (B) 10 (C) 14 (D) 4

Sol. **[C]**

Proper Method	Short Trick
$f(x) + f(x + 7) = 10$ ....(1) replace $x \to x + 7$ $f(x + 7) + f(x + 14) = 10$ .... (2) subtracting (2) from (1) $f(x) - f(x + 14) = 0$ $f(x) = f(x + 14)$ $\therefore T = 14$	$T = 2\|b - a\| = 2\|7 - 0\| = 14$

Q.2 If $f(x + 1) + f(x + 8) = 5$, then period of function $f(x)$ is -
(A) 14 (B) 7 (C) 5 (D) 1

Sol. **[A]**

Proper Method	Short Trick
$f(x + 1) + f(x + 8) = 5$ .... (1) replace $x \to x + 7$ $f(x + 8) + f(x + 15) = 5$ .... (2) subtract (2) from (1) $\Rightarrow f(x + 1) - f(x + 15) = 0$ $\Rightarrow f(x + 1) = f(x + 15)$ replace $x \to x - 1$ $\Rightarrow f(x) = f(x + 14)$ $\therefore T = 14$	$T = 2\|b - a\| = 2\|8 - 1\| = 14$

Trick -2 $[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + ... + \left[x + \frac{n-1}{n}\right] = [nx]$, $n \in N$,
(where $[\cdot]$ represents greatest integer function)

Q.3 The value of $\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{1000}\right] + \left[\frac{1}{2} + \frac{2}{1000}\right] + \left[\frac{1}{2} + \frac{3}{1000}\right] + ... + \left[\frac{1}{2} + \frac{999}{1000}\right]$ is equal to (where $[\cdot]$ represents greatest integer function) -
(A) 1000 (B) 250 (C) 500 (D) 999

Sol. **[C]**

Proper Method	Short Trick
$\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{1000}\right] + \left[\frac{1}{2} + \frac{2}{1000}\right] + ... + \left[\frac{1}{2} + \frac{500}{1000}\right]$ $+ \left[\frac{1}{2} + \frac{501}{1000}\right] + \left[\frac{1}{2} + \frac{502}{1000}\right] + ... + \left[\frac{1}{2} + \frac{999}{1000}\right]$ $= \underbrace{(0 + 0 + 0.... + 0)}_{500 \text{ terms}} + \underbrace{(1 + 1 + 1.... + 1)}_{500 \text{ terms}}$ $= 500$	$= [nx] = \left[1000 \times \frac{1}{2}\right] = 500$

Trick -3 Range of function $f(x)=\frac{ax+b}{cx+d}, x\neq -\frac{d}{c}$ is $R-\left\{\frac{a}{c}\right\}$

Q.4 The range of function $f(x)=\frac{3x-2}{4x+5}, x\neq -\frac{5}{4}$ is -

(A) R (B) $R-\left\{\frac{3}{4}\right\}$ (C) R^- (D) $R-\left\{\frac{2}{5}\right\}$

Sol. **[B]**

Proper Method	**Short Trick**
Let $y=\frac{3x-2}{4x+5}$ $\Rightarrow 4xy+5y=3x-2$ $\Rightarrow 4xy-3x=-5y-2$ $\Rightarrow x(4y-3)=-(5y+2)$ $\Rightarrow x=-\frac{(5y+2)}{4y-3}$ $\Rightarrow 4y-3\neq 0$ $\Rightarrow y\neq \frac{3}{4}$ $\therefore$ Range $R-\left\{\frac{3}{4}\right\}$	Range = $R-\left\{\frac{3}{4}\right\}$

Q.5 The range of the function $f(x)=\frac{3x+5}{2x-1}, x\neq \frac{1}{2}$ is -

(A) $R-\left\{\frac{3}{2}\right\}$ (B) R^+ (C) R (D) R_0

Sol. **[A]**

Proper Method	**Short Trick**
$y=\frac{3x+5}{2x-1}$ $\Rightarrow 2xy-y=3x+5$ $\Rightarrow 2xy-3x=y+5$ $\Rightarrow x(2y-3)=y+5$ $\Rightarrow x=\frac{y+5}{2y-3}$ $\Rightarrow 2y-3\neq 0$ $\Rightarrow y\neq \frac{3}{2}$ $\therefore$ Range $R-\left\{\frac{3}{2}\right\}$	Range = $R-\left\{\frac{3}{2}\right\}$

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